



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\sqrt{x^2 + y^2} = r, \text{ and } y/x = \tan \theta.$$

We have the following equations:

For the cone, $z = k \cot \alpha$, and for the oblique plane, $z = h - x \tan \beta x$. Eliminating z , we have for the projection on the xy -plane of the oblique section,

$$r = \frac{h \tan \alpha}{1 + \tan \alpha \tan \beta \cos \theta}.$$

If dA is an element of area in the xy -plane, the element of the conical surface of which it is the projection is $\csc \alpha \cdot dA$. Hence $dS_2 = \csc \alpha r d\theta dr$, and

$$\begin{aligned} S_2 &= \frac{1}{2} h^2 \csc \alpha \tan^2 \alpha \int_0^{2\pi} \frac{d\theta}{(1 + \tan \alpha \tan \beta \cos \theta)^2} \\ &= \frac{h^2}{2} \sec \alpha \tan \alpha \int_0^{2\pi} (1 + k \cos \theta)^{-2} d\theta, \end{aligned}$$

where

$$k = \tan \alpha \tan \beta.$$

$$\begin{aligned} \int_0^{2\pi} (1 + k \cos \theta)^{-2} d\theta &= \int_0^{2\pi} (1 - 2k \cos \theta + 3k^2 \cos^2 \theta - \dots) d\theta \\ &= 2\pi \left[1 + \frac{3}{2} k^2 + \frac{15}{8} k^4 + \frac{35}{16} k^6 + \dots \right], \end{aligned}$$

where the general term is

$$T_{n+1} = (2n+1)k^{2n} \int_0^{2\pi} \cos^{2n} \theta d\theta = \frac{(2n+1)k^{2n}(2n-1)(2n-3)\cdots 1}{2n(2n-2)\cdots 2} \frac{\pi}{2} \times 4.$$

Hence,

$$T_{n+1} = T_n \times \frac{2n+1}{2n} k^2,$$

where

$$k = \tan \alpha \tan \beta.$$

$$T_1 = \pi k^2 \tan \alpha \sec \alpha = \pi a^2 \csc \alpha = S_1.$$

Also solved by ALBERT N. NAUER.

371. Proposed by B. F. FINKEL, Drury College.

Prove that the shortest distance between two curves or surfaces is normal to each.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Let us prove this for two surfaces; the proof for two curves is similar. Suppose the equations of the surfaces are $f(x, y, z) = 0$, $F(x, y, z) = 0$, and that the shortest distance is between the points (x_1, y_1, z_1) on f and (x_2, y_2, z_2) on F . Then we wish to make $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$ a minimum with the auxiliary conditions $f(x_1, y_1, z_1) = 0$, $F(x_2, y_2, z_2) = 0$. Necessary conditions are that the partial derivatives of

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 + \lambda f(x_1, y_1, z_1) + \mu F(x_2, y_2, z_2),$$

where λ and μ are constants, shall all vanish. This gives us six equations, of which these are types:

$$2(x_1 - x_2) + \lambda \frac{\partial f}{\partial x_1} = 0,$$

$$-2(x_1 - x_2) + \mu \frac{\partial F}{\partial x_2} = 0.$$

From these, we have at once

$$\frac{\partial f}{\partial x_1} : \frac{\partial f}{\partial y_1} : \frac{\partial f}{\partial z_1} = x_1 - x_2 : y_1 - y_2 : z_1 - z_2.$$

But $\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial z_1$, are proportional to the direction cosines of the normal to $f(x_1, y_1, z_1) = 0$ at (x_1, y_1, z_1) . And $x_1 - x_2, y_1 - y_2, z_1 - z_2$, are proportional to the direction cosines of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) . Hence this line coincides with the normal. Similarly it is normal to $F(x_1, y_1, z_1) = 0$. Hence, the shortest distance must be perpendicular to both surfaces, *but not necessarily conversely*.

372. Proposed by V. M. SPUNAR, Chicago, Illinois.

Find the condition that the equation

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \left(1 + \frac{a^2}{x^2}\right)y = 0$$

should have one solution expressible in integral powers of x ; and show that if this condition is satisfied, every other solution of the equation possesses a logarithmic infinity at the origin.

SOLUTION BY ALBERT N. NAUER, St. Louis, Mo.

Let $y = vx^{-\frac{1}{2}}$. Then finding the first and second derivatives of y with respect to x and substituting these values in the equation, we have

$$\frac{d^2v}{dx^2} - v = \left(\frac{a^2 - \frac{1}{4}}{x^2}\right)v.$$

This equation is of the form

$$\frac{d^2u}{dx^2} - \alpha^2 u = \frac{m(m+1)}{x^2}u,$$

where $m(m+1) = a^2 - \frac{1}{4}$, or $m = -\frac{1}{2} \pm a$ and $\alpha = 1$. This equation has the three following known solutions:

$$(1) \quad u = x^{-m} \left(1 - \frac{1}{m - \frac{1}{2}} \frac{\alpha^2 x^2}{2^2} + \frac{1}{(m - \frac{1}{2})(m - \frac{3}{2})} \frac{\alpha^4 x^4}{2! 2^4} - \frac{1}{(m - \frac{1}{2})(m - \frac{5}{2})(m - \frac{7}{2})} \frac{\alpha^6 x^6}{3! 2^6} + \dots \right),$$

$$(2) \quad u = e^{\alpha x} x^{-m} \left(1 - \frac{m}{m} \alpha x + \frac{m(m-1)}{m(m - \frac{1}{2})} \frac{\alpha^2 x^2}{2!} + \frac{m(m-1)(m-2)}{m(m - \frac{1}{2})(m-1)} \frac{\alpha^3 x^3}{3!} - \dots \right),$$

$$(3) \quad u = e^{-\alpha x} x^{-m} \left(1 + \frac{m}{m} \alpha x + \frac{m(m-1)}{m(m - \frac{1}{2})} \frac{\alpha^2 x^2}{2!} - \frac{m(m-1)(m-2)}{m(m - \frac{1}{2})(m-1)} \frac{\alpha^3 x^3}{3!} + \dots \right).$$